Nonlinear Kalman filter phase unwrapping algorithm based on the terrain

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Abstract

Phase unwrapping is one of the key steps in InSAR data processing. In condition of steep terrain, excessive stripes reduce the conventional phase unwrapping algorithms' implement, resulting in error propagation of unwrapping phase. Therefore, this paper presented a nonlinear Kalman filter phase unwrapping algorithm based on terrain. This algorithm uses local fringe frequency estimation as input control variable. Chirp-Z transform is added into Fourier transform in local frequency estimation, and this improves the unwrapping result accuracy. Results obtained with simulated and real data validate effectiveness of proposed method through analyzing and comparing with Kalman filter method, nonlinear Kalman filter method and quality map guiding method.

Keywords: InSAR, nonlinear Kalman filter, phase unwrapping, Chirp-Z transform, terrain

1 Introduction

One of the important steps in InSAR data processing is phase unwrapping, its accuracy will directly influence height measurement. We refer to the book by Ghiglia and Pritt [1] for an excellent overview. Traditionally, there have been two general types of conventional phase unwrapping methods. The algorithms of the first group, generally named path-following algorithms [2-4]. Those advantages are that computing speed is fast and demanding memory is less. These algorithms isolate problematic zones containing residues and can be unwrapped the interferogram by avoiding these zones. The techniques of the second group provide a global solution which minimizes a cost function over the whole interferogram [5-8]. Independently from this traditional classification, some techniques make use of a prefiltering stage before starting unwrapping procedure with filtered phase, for instance [9]. Conventional Kalman filter algorithm transforms phase unwrapping into state estimation problem, through the establishment of phase state space model and vector observation equation, and Kalman filter is used to unwrap and filter simultaneously. However, the layover phenomenon makes discontinuous streaks appear in course of phase unwrapping; coherence reduction also affects reliability of DEM, which occurs in the urban or dried area; in steep terrain, too many stripes reduce phase unwrapping algorithm's execution, and this makes error propagation phenomenon appear in unwrapping results. These disadvantages will affect accuracy and resolution of image which generated. But the previous two shortcomings are difficult to overcome, in this paper nonlinear Kalman filter phase unwrapping algorithm based on terrain is proposed. It can be implemented through introduction of input control variable associated with terrain to the state space model, then in iterative formula Chirp-Z transform is added into Fourier transform to estimate local frequency, accuracy of phase unwrapping is improved.

2 Nonlinear Kalman filter phase unwrapping algorithm based on terrain

2.1 OBSERVATION EQUATION

Inphase and quadrature components of complex interferogram are consisted as two noisy observations of true interferometric phase. Here, substituting again m, n pixel by k pixel, $\phi(k)$ represents real value of interfergram phase at k pixel, observation equation can be written as follows [10]:

$$y(k) = h[\phi(k)] + v(k) = \begin{Bmatrix} \dfrac{\mathrm{Im}[z(k)]}{a(k)} \\ \dfrac{\mathrm{Re}[z(k)]}{a(k)} \end{Bmatrix} = \begin{Bmatrix} \sin(\phi(k)) \\ \cos(\phi(k)) \end{Bmatrix} + \begin{Bmatrix} v_1(k) \\ v_2(k) \end{Bmatrix}, \quad (1)$$

where, $z(k)$ is complex interferogram, $a(k)$ represents amplitude of interferogram, $h(\cdot)$ represents non-linearized mapping between $y(k)$ and state vector $\phi(k)$. $v_1(k)$ and $v_2(k)$ are assumed to be white Gaussian noise, then:

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$$E\{v(k)\}=0; v(k)=\begin{Bmatrix} v_1(k) \\ v_2(k) \end{Bmatrix},$$

$$R(k)=E\left\{v(k)v(j)^T\right\}=diag\left[\sigma_v^2(k)\delta(k,j)\right], \quad (2)$$

where, $\delta_v^2(k)=\frac{1}{SNR(k)}$, $SNR(k)$ represents SNR at k pixel, $\delta_v^2(k)=\frac{1}{SNR(k)}$ is Kronecker function, and $\delta(k,j)=\begin{Bmatrix} 1, k=j \\ 0, k\neq j \end{Bmatrix}$.

2.2 STATE SPACE MODEL

Considering the terrain factors on phase unwrapping, control variables associated with terrain is added, and then improved nonlinear Kalman filter state space model is:

$$\begin{cases} x(k+1)=x(k)+D\varsigma+u(k) \\ u(k)=\hat{u}(k)+w(k) \end{cases},$$

$$E\{w(k)\}=0; E\{w(k)w(j)\}=Q(k)\delta(k,j), \quad (3)$$

where $u(k)$is actual phase gradient, $\hat{u}(k)$represents phase gradient estimation determined by complex phase, $w(k)$ represents unknown estimation error, $Q(k)$ is noise covariance matrix, D is the input control matrix, ς represents control input variable related with terrain, here is the local estimated frequency.

2.3 PHASE UNWRAPPING ALGORITHM BASED ON TERRAIN

Original nonlinear Kalman filter phase unwrapping algorithm estimate local frequency using the Fourier transform. In nonlinear Kalman filter phase unwrapping algorithm based on the terrain, using Chirp-Z Transform added into two-dimensional Fourier Transform algorithm is applied to estimate local frequency of interferogram fringe, accuracy of phase unwrapping is improved. Here, choosing a two-dimensional Fourier transform at 32×32 point is added to Chirp-Z transform at point 128×128 to obtain local frequency. Nonlinear Kalman filter algorithm based on terrain's procedure is as follows:

The first step, according to (4), it calculates Sigma estimation value $\chi_i^-(k+1)$ at $k+1$ pixel, state predictive value and its corresponding error covariance $P_{xx}^-(k+1)$ are:

$$\chi_i^-(k+1)=\sum d(k)\chi_i^-(k),$$

$$x(k)=\sum_{i=0}^{2n} W_i^m \chi_i^-(k+1)+\sum d(k)\varsigma(k),$$

$$P_{xx}^-(k+1)=\sum_{i=0}^{2n} W_i^0\left[\chi_i^-(k+1)-\bar{x}(k+1)\right]\times$$
$$\left[\chi_i^-(k+1)-x(k+1)\right]^T+\sum d(k)Q(k), \quad (4)$$

where, $\varsigma(k)$ is local frequency estimation at k pixel. Solution can be obtained from Equation (5), and window size is selected as the size 3×3:

$$J(f_y,f_x)=$$
$$\left|\sum_{y=k_m-(D_k-1)/2}^{k_m+(D_k-1)/2}\ \sum_{x=k_n-(D_l-1)/2}^{k_n+(D_l-1)/2} x_n(y,x)\exp(-j2\pi(f_y k_n+f_x k_m))\right|, \quad (5)$$

where, f_x and f_y are respectively range and azimuth local frequency estimation with (a, s) pixel as the center. It computes weight $d(k)$ from Equation (6):

$$d(k)=[P_{xx}(k)]^{-1}g(k)/\sum[P_{xx}(k)]^{-1}g(k),$$

$$g(k)=\begin{Bmatrix} 0, k\ pixel\ is\ wrapped\ phase \\ 1, k\ pixel\ is\ unwrapped\ phase \end{Bmatrix}. \quad (6)$$

The second step, it calculates the predicted Sigma value $\xi_i^-(k+1)$, observed value $y^-(k+1)$ and its corresponding error covariance $P_{yy}^-(k+1)$:

$$P_{yy}^-(k+1)=\sum_{i=0}^{2n} W_i^{(c)}\left[\xi_i^-(k+1)-y^-(k+1)\right]\times$$
$$\left[\xi_i^-(k+1)-y^-(k+1)\right]^T+R(k+1),$$

$$\xi_i^-(k+1)=h[\chi_i^-(k+1)],$$

$$y^-(k+1)=\sum_{i=0}^{2n} W_i^{(m)}\xi_i^-(k+1). \quad (7)$$

The third step, it calculates unwrapped phase value $x(k+1)$ and its corresponding estimation error covariance $P_{xx}(k+1)$:

$$P_{xy}^-(k+1)=$$
$$\sum_{i=0}^{2n} W_i^c[\chi_i^-(k+1)-x^-(k+1)]\times[\xi_i^-(k+1)-y^-(k+1)]^T,$$

$$J(k+1)=P_{xy}^-(k+1)/P_{yy}^-(k+1)$$

$$x(k+1)=x^-(k+1)+J(k+1)[y(k+1)-y^-(k+1)]$$
$$P_{xx}(k+1)=P_{xx}^-(k+1)-J(k+1)P_{xy}^-(k+1)J(k+1)^T, \quad (8)$$

where, $J(k+1)$ is gain matrix at $k+1$ pixel, $P_{xy}^-(k+1)$ is prediction observed value covariance matrix at $k+1$ pixel.

According to the above three steps, nonlinear Kalman filter based on terrain will process noise filter and phase unwrapping simultaneously along a particular path.

3 Experimental results and analysis

3.1 SIMULATED DATA EXPERIMENT

In order to verify validity of nonlinear Kalman filter algorithm based on terrain, this paper first uses size of 100×100 pixels simulated interferogram to do experiment (Figure 1), and its terrain is relatively steep. 0.39dB of SNR is added into simulated data, shown in Figure 2.As can be seen from Figure 3, nonlinear Kalman filter algorithm based on terrain and nonlinear Kalman filter algorithm have roughly the same as the original phase diagram, but nonlinear Kalman filter algorithm's result is more rough; only nonlinear Kalman filter algorithm based on terrain's unwrapping result is more reliable; Kalman filter algorithm's and quality guided algorithm's results are quite different from original phase diagram.

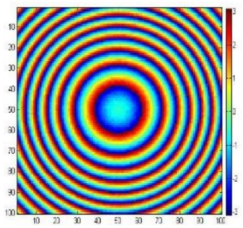


FIGURE 1 Interferogram without noise

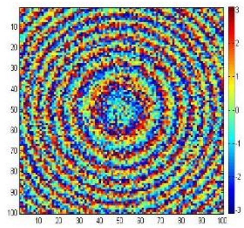
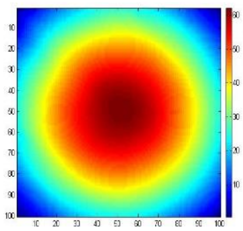


FIGURE 2 Interferogram with noise



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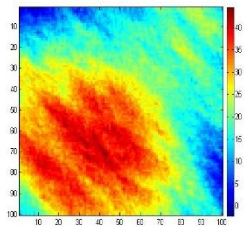

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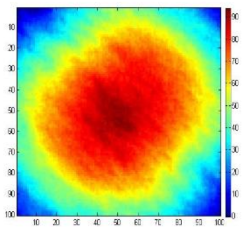
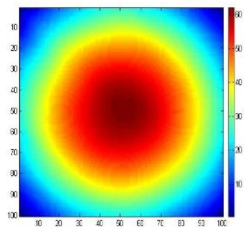


c)



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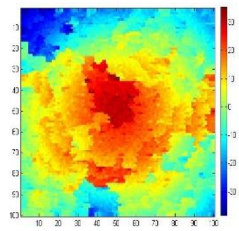
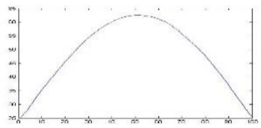


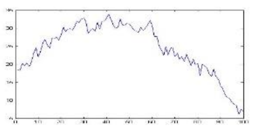
e)

FIGURE 3 Unwrapping result: a) Real phase diagram; b) Kalman filter algorithm, c) Nonlinear Kalman filter algorithm, d) Nonlinear Kalman filter algorithm based on based on terrain, e) Quality guided algorithm

Figure 4 is sectional figure of four unwrapping algorithms. Nonlinear Kalman filter algorithm based on terrain's peak reaches the maximum of real one. Accuracy of the result is higher; maximum value of nonlinear Kalman filter is far more than real one's. The curve is more rough, and unwrapping result's accuracy is poorer; Kalman filter algorithm's and quality guided algorithm's maximum values are far less than real one's. Accuracy of unwrapping results are the worst.



a)



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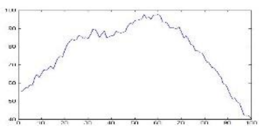
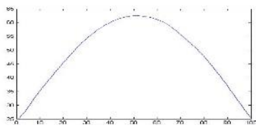


c)



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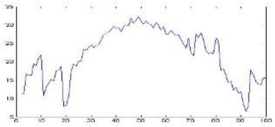


e)

FIGURE 4 Unwrapping phase the 50th line section results: a) Real phase diagram; b) Kalman filter algorithm, c) Nonlinear Kalman filter algorithm, d) Nonlinear Kalman filter algorithm based on based on terrain, e) Quality guided algorithm

3.2 REAL DATA EXPERIMENT

Two ALOS PALSAR images at January 10, 2009 and February 25, 2009 in Beijing are selected as primary and secondary image, part of interferogram (Figure 5) is selected after registration, interference, and ground effect. The 100×100 pixels area's terrain is more complicate and steeper, and Figure6 is the coherence map corresponding to it. As can be seen from Figure7, Kalman filter algorithm and nonlinear Kalman filter algorithm all have more reliable results, but those unwrapping results are not smooth than nonlinear Kalman filter algorithm based on terrain's. It can restore relatively smooth phase surface. But quality guided algorithm can not show low terrain at left corner, high terrain at right corner surface, and its unwrapping result is not reliable.

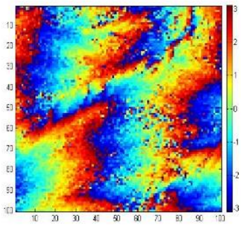


FIGURE 5 Interferogram

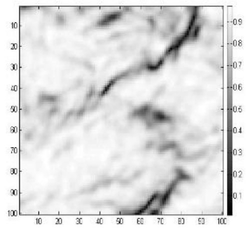
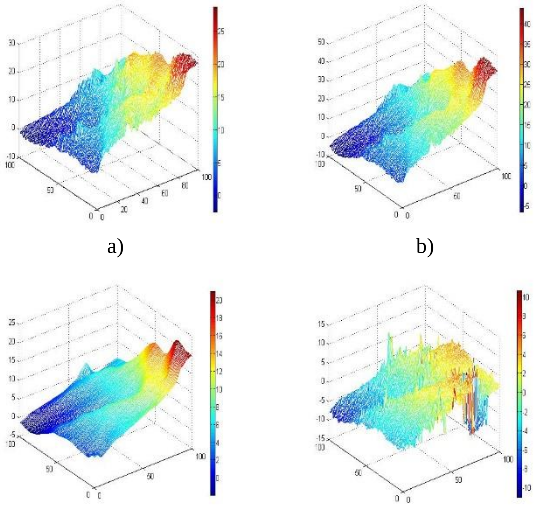


FIGURE 6 Coherence map



a) b) c) d)

FIGURE 7 Unwrapping results: a) Kalman filter method; b) Nonlinear Kalman filter method; c) Nonlinear Kalman filter method based on topographic factors; d) Quality guided method

From Table 1 we can see, discontinuous point's number of quality guided method is the largest and ε value is the maximum, so this shows that many phase transition points are generated in the process of phase unwrapping, serious error propagation phenomena is appeared. Discontinuous points' number of nonlinear Kalman filter algorithm based on terrain is 0, and ε value is the minimum, this indicates unwrapping result's quality is the highest. The discontinuous points' number of the other two methods is smaller, and ε value is smaller, this shows the performance of anti-phase distortion is better.

TABLE 1 ε value and discontinuous points' number in four methods

Unwrapping methods	Discontinuous points' number	ε value
Kalman filter method	97	1.5974
Nonlinear Kalman filter method	63	1.4718
Quality guided method	292	1.8618
Nonlinear Kalman filter method based on terrain	0	1.2886

TABLE 2 Difference between rewrapped results of four methods and original wrap phase

Unwrapping methods	Average error	Intermediate error	Root mean square
Kalman filter method	1.9437	3.3520	1.4580
Nonlinear Kalman filter method	1.3992	2.9950	1.3941
Quality guided method	1.0197	2.5747	0.4447
Nonlinear Kalman filter method based on terrain	0.0363	0.3507	0.3466

From Table 2 we can see, difference of four methods is not big in error absolute value's maximum aspect， and only quality guided method and nonlinear Kalman filter method are smallest in error absolute minimum aspect. In the mean error, intermediate error and root mean square aspect, nonlinear Kalman filter method based on terrain is the smallest, unwrapping accuracy is highest, and Kalman filter method's is higher, unwrapping result's reliability is less.

4 Conclusions

In this paper, local error propagation caused by steep terrain which makes unwrapping result inaccurate, nonlinear Kalman filter phase unwrapping algorithm based on terrain is proposed. Through introducing input control terrain variable in the state space model, then using two-dimensional Chirp-Z transform added into Fourier transform to estimate local frequency, accuracy of phase unwrapping is improved. Unwrapping results of simulated and real data verify that the algorithm is reliable and effective. Because in practice the error is generated during image processing, how to further take into account the model error, making the unwrapping result more close to the real phase is need to further study.

Acknowledgments

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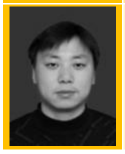

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